



Secondary Mathematics Subject Knowledge Audit KS3/KS4 2017-2018

Name

Number KS3	Familiar with	Competent to teach	Planned, researched, resourced	Date taught
Understand and use place value for decimals, measures and integers of any size				
Order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥				
Use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property				
Use the 4 operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative				
Use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals				
Recognise and use relationships between operations including inverse operations				
Use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations				
Interpret and compare numbers in standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or negative integer or 0				
Work interchangeably with terminating decimals and their corresponding fractions				
Define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express 1 quantity as a percentage of another, compare 2 quantities using percentages, and work with percentages greater than 100%				

Interpret fractions and percentages as operators				
Use standard units of mass, length, time, money and other measures, including with decimal quantities				
Round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]				
Use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \leq b$				
Use a calculator and other technologies to calculate results accurately and then interpret them appropriately				
Appreciate the infinite nature of the sets of integers, real and rational numbers				

Number KS4	Familiar with	Competent to teach	Planned, researched, resourced	Date taught
Apply systematic listing strategies, including use of the product rule for counting				
Estimate powers and roots of any given positive number				
Calculate with roots, and with integer and fractional indices				
Calculate exactly with fractions, surds and multiples of π ; simplify surd expressions involving squares				
Calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer				
Change recurring decimals into their corresponding fractions and vice versa				
Identify and work with fractions in ratio problems				
Apply and interpret limits of accuracy when rounding or truncating, including upper and lower bounds				

Algebra KS3	Familiar with	Competent to teach	Planned, researched, resourced	Date taught
Use and interpret algebraic notation				
Substitute numerical values into formulae and expressions, including scientific formulae				
Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors				
Simplify and manipulate algebraic expressions to maintain equivalence by: collecting like terms, multiplying a single term over a bracket, taking out common factors, expanding products of two or more binomials				
Understand and use standard mathematical formulae; rearrange formulae to change the subject				
Model situations or procedures by translating them into algebraic expressions or formulae and by using graphs				
Use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)				
Work with coordinates in all four quadrants				
Recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane				
Interpret mathematical relationships both algebraically and graphically				
Reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically				
Use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations				
Find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs				
Generate terms of a sequence from either a term-to-term or a position-to-term rule				
Recognise arithmetic sequences and find the n th term				
Recognise geometric sequences and appreciate other sequences that arise.				

Algebra KS4	Familiar with	Competent to teach	Planned, researched, resourced	Date taught
Simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions) by: <ul style="list-style-type: none"> • Factorising quadratic expressions • Simplifying expressions involving sums, products and powers, including the laws of indices 				
Know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments and proofs				
Where appropriate, interpret simple expressions as functions with inputs and outputs; interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'				
Use the form $y=mx+c$ to identify parallel and perpendicular lines; find the equation of the line through two given points, or through one point with a given gradient				
Identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically and turning points by completing the square				
Recognize, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function, the exponential functions and the trigonometric functions				
Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration				
Calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts				
Recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point				

Solve quadratic equations including those that require rearrangement algebraically by factorising, by completing the square and by using the quadratic formula; find approximate solutions using a graph				
Solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically; find approximate solutions using a graph				
Find approximate solutions to equations numerically using iteration				
Translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution				
Solve linear inequalities in one or two variables, and quadratic inequalities in one variable; represent the solution set on a number line, using set notation and on a graph				
Recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions				
Deduce expressions to calculate the nth term of linear and quadratic sequences.				

Ratio, Proportion and Rates of	Familiar	Competent	Planned,	Date
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Change KS3	with	to teach	researched, resourced	taught
Change freely between related standard units [for example time, length, area, volume/capacity, mass]				
Use scale factors, scale diagrams and maps				
Express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1				
Use ratio notation, including reduction to simplest form				
Divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio				
Understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction				
Relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions				
Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics				
Solve problems involving direct and inverse proportion, including graphical and algebraic representations				
Use compound units such as speed, unit pricing and density to solve problems.				

Ratio, Proportion and Rates of	Familiar	Competent	Planned,	Date
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Change KS4	with	to teach	researched, resourced	taught
Compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)				
Convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts				
Understand inverse proportion and construct and interpret equations that describe direct and inverse proportion				
Interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion				
Interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts				
Set up, solve and interpret the answers in growth and decay problems, including compound interest and work with general iterative processes.				

Geometry and Measures KS3	Familiar	Competent	Planned,	Date
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	with	to teach	researched, resourced	taught
Derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)				
Calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes				
Derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line				
Describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric				
Use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles				
Derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies				
Identify properties of, and describe the results of, translations, rotations and reflections applied to given figures				
Identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids				
Apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles				
Understand and use the relationship between parallel lines and alternate and corresponding angles				
Derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons				

Apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs				
Use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles				
Use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D				
Interpret mathematical relationships both algebraically and geometrically.				

Geometry and Measures KS4	Familiar with	Competent to teach	Planned, researched,	Date taught
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			resourced	
Interpret and use fractional and negative scale factors for enlargements				
Describe the changes and invariance achieved by combinations of rotations, reflections and translations				
Identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment				
Apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results				
Construct and interpret plans and elevations of 3D shapes				
Interpret and use bearings				
Calculate arc lengths, angles and areas of sectors of circles				
Calculate surface areas and volumes of spheres, pyramids, cones and composite solids				
Apply the concepts of congruence and similarity, including the relationships between lengths, areas and volumes in similar figures				
Apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles and, where possible, general triangles in two and three dimensional figures				
Know the exact values of $\sin \theta$ and $\cos \theta$ for 0° , 30° , 45° , 60° and 90° ; know the exact value for $\tan \theta$ for 0° , 30° , 45° and 60°				
Know and apply the sine rule, , and cosine rule to find unknown lengths and angles				
Know and apply $\text{Area} = \frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle				
Describe translations as 2D vectors				
Apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; use vectors to construct geometric arguments and proofs				

Probability KS3	Familiar	Competent	Planned,	Date
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	with	to teach	researched, resourced	taught
Record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale				
Understand that the probabilities of all possible outcomes sum to 1				
Enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams				
Generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities.				

Probability KS4	Familiar with	Competent to teach	Planned, researched, resourced	Date taught
Apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one				
Use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size				
Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions				
Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams.				

Statistics KS3	Familiar	Competent	Planned,	Date
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	with	to teach	researched, resourced	taught
Describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)				
Construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data				
Describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs.				

Statistics KS4	Familiar with	Competent to teach	Planned, researched, resourced	Date taught
Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling				
Interpret and construct tables and line graphs for time series data				
Construct and interpret diagrams for grouped discrete data and continuous data				
Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through; appropriate graphical representation involving discrete, continuous and grouped data, including box plots, appropriate measures of central tendency and spread including quartiles and inter-quartile range				
Apply statistics to describe a population				
Use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.				

Working Mathematically

In addition to the content in this booklet, you should begin to feel confident ensuring lessons will allow students to work mathematically allowing students to develop their fluency, mathematical reasoning and problem solving skills.

The national curriculum for mathematics aims to ensure that all pupils:

- *become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.*
- ***reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language*
- *can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.*

(National Curriculum POS, 2014)

Key Stage 3

Develop fluency

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]
- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics.

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Key Stage 4: Working Mathematically

Develop fluency

- consolidate their numerical and mathematical capability from key stage 3 and extend their understanding of the number system to include powers, roots {and fractional indices}
- select and use appropriate calculation strategies to solve increasingly complex problems, including exact calculations involving multiples of π {and surds}, use of standard form and application and interpretation of limits of accuracy
- consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions, {and expressions involving surds and algebraic fractions}
- extend fluency with expressions and equations from key stage 3, to include quadratic equations, simultaneous equations and inequalities
- move freely between different numerical, algebraic, graphical and diagrammatic representations, including of linear, quadratic, reciprocal, {exponential and trigonometric} functions
- use mathematical language and properties precisely.

Reason mathematically

- extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically
- extend their ability to identify variables and express relations between variables algebraically and graphically
- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments {and proofs}
- reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally
- assess the validity of an argument and the accuracy of a given way of presenting information.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts
- make and use connections between different parts of mathematics to solve problems
- model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem.